THE FINITE DIFFERENCES METHOD TO DETERMINE THE POTENTIAL FUNCTION VALUES IN THE THREE-DIMENSIONAL SPACE AROUND A SEAGOING SHIP (FINDI)

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Abstract. The purpose of the paper is to present an algorithm, based on the Finite Differences Method (FINDI), for the computation of the electric potential on the three spatial directions around a seagoing ship.

1. INTRODUCTION

The electric field of a sea-going ship assumes a slow rate variation in time, if energy conversions are negligible. The equations with partial derivates which describe the main magnitudes are of elliptical type. Thus, the study of the field phenomena is processed in the framework of the scalar potential theory.
A potential field having the divergence of the state vector equally zero in all the points P, in a domain Ω, namely:

$$(\text{div } E)_P = 0, \forall P \in \Omega \qquad (1)$$

is called a "Laplace" field in the domain Ω.
The potential function V (defined by $E = -\text{grad}V$) verifies in Ω the equation:

$$\Delta V = 0, \text{ called Laplace equation.} \qquad (2)$$

In order to solve equation (2), the boundary conditions of the problem have to be find. There are many methods specialised to solve this kind of problems. For its simplicity and efficiency, "The Finite Differences Method" suits very well to such a problem. Besides, there is available a huge amount of computer code to solve it.
The "Finite Differences Method" replaces the equations with partial derivates (describing a problems with boundary conditions) by equations with finite differences. The result of this method consists of a series of digital data, representing the potential values in the nodes of a network (applied to a chosen domain). These digital data are obtained by solving a linear

algebraic system, the discrete counterpart of the equations with partial derivates in the nodes of the network.
The accuracy of the method depends of:
- the discrete network;
- the type of discretization for the equations with partial derivates;
- the precision of approximation for the boundary conditions;
- the computation techniques (methods and equipment).
In order to approximate a Laplace equation by the "Finite Differences Method" we choose a discrete model in a three-orthogonal Cartesian coordinates system formed by cubic network.

2. THE ELECTRIC POTENTIAL FOR BOUNDARY CONDITIONS OF FIRST TYPE (DIRICHLET)

The physical measurements were picked-up on a seagoing ship of small dimensions, having the immersed bull with the configuration shown in fig.1. The potential measurements, on the boundary and inside the chosen domain, were performed offshore of the Black Sea, in the region of Mangalia city. These determinations were necessary in order to get the potential values and to use them as boundary conditions in a Dirichlet problem.
It can be notice that the studied domain stretches between the frames C39 and C47, having the dimensions from fig.1. The cross plane of the ship, determined by the plane yoz and the hull is approximated by a rectangle 3 x 1 m.

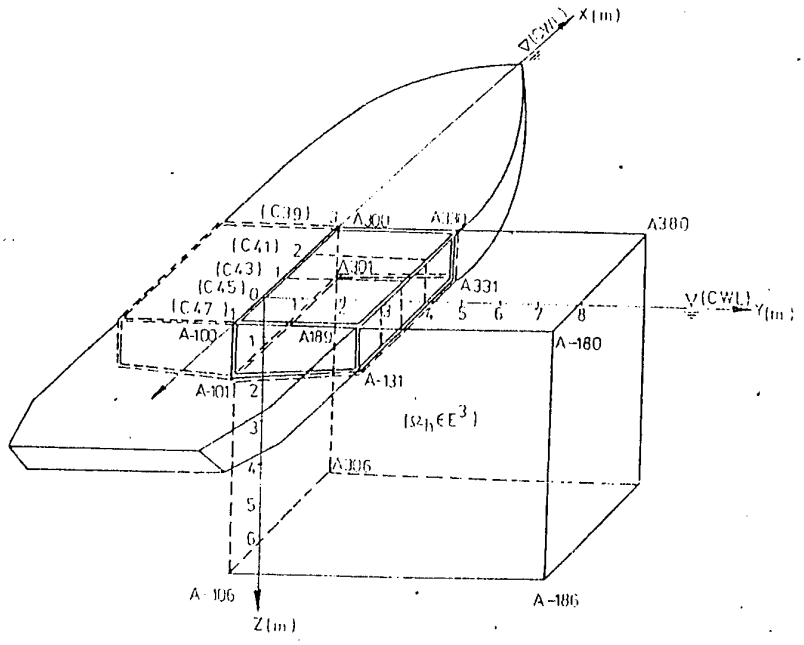

Fig.1 The immersed bull of the ship and the measure domain Ω_h

The index of any point A in fig.1 indicates the coordinates of A in the 3D space. The values of the potential V(x,y,z), inside the chosen domain, was determined by solving the differential equation (2). For this, we have used an approximation with seven inner nodes in the 3D space.

These nodes are distributed as shown in fig.2, i.e., a discrete cubic network with a characteristic structure R7.

In fig.2b it can be seen that the node A_0 (x,y,z) is surrounded by the nodes A_1 ... A_6 .The same notation is preserved for any inner node.

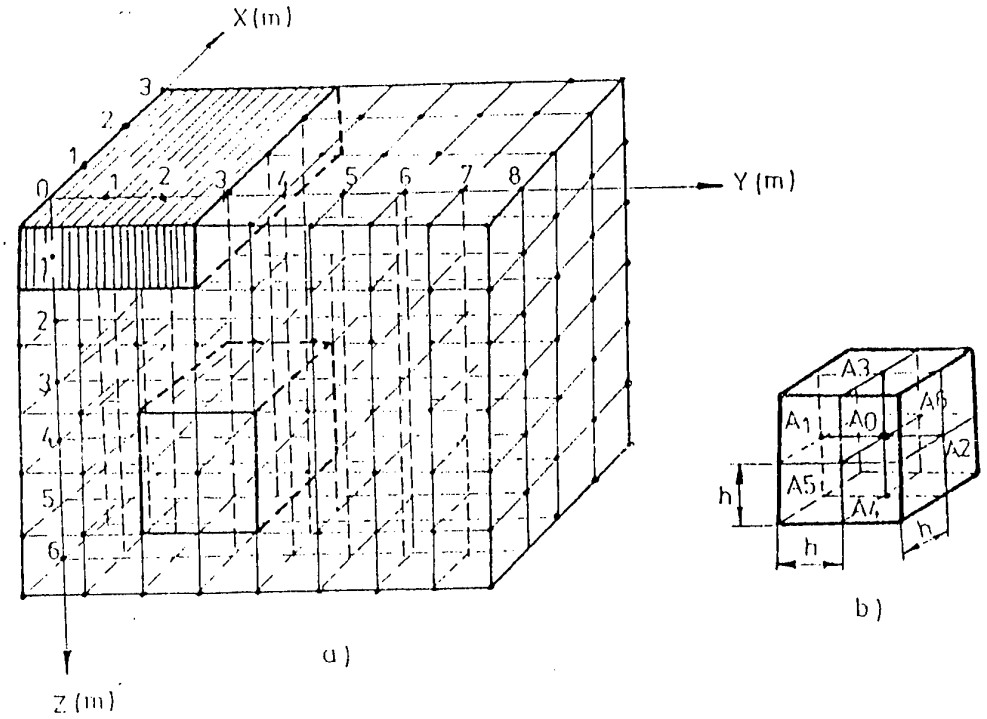

Fig. 2 The discrete cubic network :
a) the discretization of the chosen domain
b) the characteristic structure R7

In the case of first type boundary conditions the nodes A_0 will be located only inside the chosen domain . If we develop the potential V (x,y,z) after Taylor formula, around A_0 (x,y,z), the result is:

$$V(x_0+h,\ y_0+h,\ z_0+h) = V(x_0,\ y_0,\ z_0) + \sum 1/m!\cdot[(\partial V/\partial x)_{A0}h+(\partial V/\partial y)_{A0}h+(\partial V/\partial x)_{A0}\cdot h +$$
$$+ (\partial V/\partial x)_{A0}h]^m = V_0 + (\partial V/\partial x+\partial V/\partial y+\partial V/\partial z)_{A0}\cdot h + 1/2!(\partial^2 V/\partial x^2+\partial^2 V/\partial y^2+\partial^2 V/\partial z^2)_{A0}\cdot h^2 +$$
$$+ 2/2!\cdot(\partial^2 V/\partial x\partial y+\partial^2 V/\partial x\partial z+\partial^2 V/\partial y\partial z)_{A0}\cdot h^2 + 1/3!(\partial^3 V/\partial x^3+\partial^3 V/\partial y^3+\partial^3 V/\partial z^3)_{A0}\cdot h^3 +$$
$$+ 3/3!(\partial^3 V/\partial x\partial y^2+\partial^3 V/\partial x\partial z^2+\partial^3 V/\partial x^2\partial y+\partial^3 V/\partial x^2\partial z+\partial^3 V/\partial z^2\partial y+\partial^3 V/\partial y^2\partial z)_{A0}\cdot h^3 + \ldots \quad (3)$$

From a practical point of view, the accuracy of the computation is satisfactory, even without taking into account the terms of above second degree. In particular, the following equations with partial derivates are successively obtained:

- for the node A_1 (x_0, y_0 - h, z_0):

$$V_1 = V_0 - (\partial V/\partial y)_{A0}\cdot h + 1/2!\cdot(\partial^2 V/\partial y^2)_{A0}\cdot h^2 - \ldots \quad (4)$$

- for the node A_2 (x_0, y_0+h, z_0):

$$V_2 = V_0 + (\partial V/\partial y)_{A0}\cdot h + 1/2!\cdot(\partial^2 V/\partial y^2)_{A0}\cdot h^2 + \ldots \quad (5)$$

- for the node A_3 $(x_0, y_0, z_0 - h)$:

$$V = V_0 - (\partial V/\partial z)_{A0}\cdot h + 1/2!\cdot(\partial^2 V/\partial z^2)_{A0}\cdot h^2 - \ldots \quad (6)$$

- for the node A_4 $(x_0, y_0, z_0 + h)$:

$$V_4 = V_0 + (\partial V/\partial z)_{A0}\cdot h + 1/2!\cdot(\partial^2 V/\partial z^2)_{A0}\cdot h^2 + \ldots \quad (7)$$

- for the node A_5 $(x_0 - h, y_0, z_0)$:

$$V_5 = V_0 - (\partial V/\partial x)_{A0}\cdot h + 1/2!\cdot(\partial^2 V/\partial x^2)_{A0}\cdot h^2 - \ldots \quad (8)$$

- for the node A_1 $(x_0 + h, y_0, z_0)$:

$$V_6 = V_0 + (\partial V/\partial x)_{A0}\cdot h + 1/2!\cdot(\partial^2 V/\partial x^2)_{A0}\cdot h^2 + \ldots \quad (9)$$

From relations 5 ÷ 10 we have:

$$V_1 + V_2 + V_3 + V_4 + V_5 + V_6 = 6\ V_0 + (\partial^2 V/\partial x^2 + \partial^2 V/\partial y^2 + \partial^2 V/\partial z^2)_{A0}\cdot h^2 = 6\ V_0 + \Delta V\cdot h^2 \quad (11)$$

Taking into account equation (2) for a Laplace field, we have:

$$V_1 + V_2 + V_3 + V_4 + V_5 + V_6 - 6\ V_0 = 0 \quad (12)$$

Similarly (for any point of the network) the following relation holds:

$$V_{x_0, y_0 - 1, z_0} + V_{x_0, y_0, z_0 - 1} + V_{x_0, y_0, z_0} + V_{x_0, y_0, z_0 + 1} + V_{x_0 - 1, y_0, z_0} + V_{x_0 + 1, y_0, z_0} - 6\ V_{x_0, y_0, z_0} = 0 \quad (13)$$

These two expressions (12 & 13) are an approximation with finite differences of the Laplace's equation ($\Delta V = 0$), in seven nodes of the 3D space; the order of the error magnitude is of zero. Equation (13) is further iterated for each inner node of the chosen domain. In particular, for the domain shown in fig.2 a system of 96 linear algebraic equations with 96 unknown variables was found out. By solving the system the digital approximation of the electric potential function V(x, y, z), x, y, z $\in \Omega_h$ is obtained.

3. CONCLUSIONS

The computation of the potential values on the entire immersed hull of a sea-going ship or in a volume around it, is a problem of considerable concern (e.g. to obtain information about the electrochemical corrosion gradient).
The proposed method has two stages as following:

- an experimental stage - the measured of the potential values, which will form the boundary conditions of Dirichlet type problem for the following computation stage, and will validate the computation as well
- a computation stage - to determine the potential values inside the considered volume

REFERENCES

[1] Aubin J. P. - "Approximation of elliptic boundary value problems" - Wiley Interscience, New York 1972

[2] Ciarlet P.G. - "Numerical analysis of the finite element method for elliptic boundary value problems" - North-Holland, Amsterdam 1977

[3] Collatz L. - "The numerical treatment of differential equations", Springer - Verlag, Berlin, Heidelberg, New York, 1966

[4] Dumitrescu I. - " Simularea cimpurilor potentiale", Romanian Academy Edition, Bucharest 1983

[5] Greenspan D. - "Introductory numerical analysis of elliptic boundary value problems" Harper and Row, New York 1965

[6] Milne W.E. - "Numerical solution of differential equations", John Wiley and Sons, New York 1953

[7] Racoveanu N. , Dodescu Gh. , Mincu I. - "Metode numerice pentru ecuatii cu derivate partiale de tip hiperbolic" - Technical Edition, Bucharest 1976

[8] Smith G.D. - Numerical solution of partial derivate equations", Oxford University Press, London 1965

[9] Susanu T. , Calueanu D. , Lingvay I. , Alexa M. - "Potential measurements on the hull of a seagoing ship. Methods and results." - The International Conference "Optim 91", "Transilvania" University, Brasov , Oct. 10 - 13, 1991

[10] x x x "Numerical solution of partial differential equations", New York, Academic Press, 1967